Gravitational Waves from Inflation with CMB-S4

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Inflation

The simplest system leading to a sufficiently long phase of accelerated expansion (that ends) is

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R \\ - \int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\right)$$

with $M_P^2\left(\frac{V'}{V}\right)^2 \ll 1$ and $M_P^2\left|\frac{V''}{V}\right| \ll 1$ somewhere

If the scalar field is nearly homogeneous, and at a position in field space such that the potential energy dominates its energy density, this leads to nearly exponential expansion.

Inflation

The simplest models of inflation predict

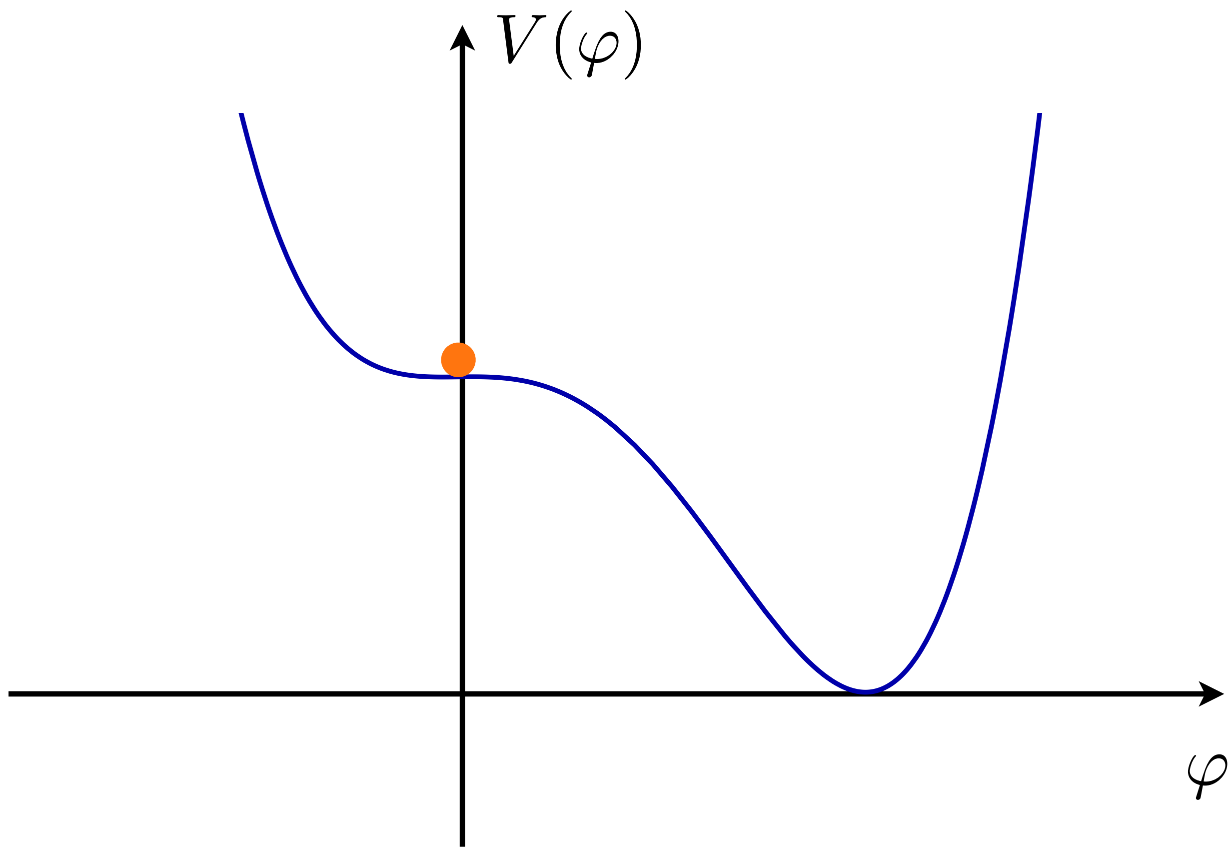
- primordial fluctuations that are
  - nearly scale invariant
  - well approximated by a power law
  - dominated by density fluctuations with subdominant contribution from gravitational waves
  - close to Gaussian
  - adiabatic
- a spatially flat universe

in excellent agreement with observations

Expectations

Inflation could be accidental

$$V(\varphi) = V_0 \left(1 + c_1\varphi + \frac{c_2}{2}\varphi^2 - \varphi^3 + \ldots\right) \quad \textbf{with} \quad c_1, c_2 \ll 1$$



Expectations

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Sufficiently long inflation only for $|c_1| \lesssim 10^{-3}$

Then

$$r \approx 8c_1^2 \lesssim 10^{-5}$$

Unobservably small gravitational wave signal

$$n_\mathrm{s} \approx 1 + 2c_2$$

No prediction for the spectral index.

Requires tuning and the observed value of spectral index is an accident.

Expectations

Instead, taking the observed value of the spectral index seriously, we might expect

$$n_s(\mathcal{N}) - 1 = -\frac{p+1}{\mathcal{N}}$$

In general (provided $\epsilon \ll 1$)

$$\frac{d\ln\epsilon}{d\mathcal{N}} - (n_{\rm s}(\mathcal{N}) - 1) - 2\epsilon = 0$$

or

$$\epsilon(\mathcal{N}) = \frac{p}{2\mathcal{N}} \frac{1}{1 \pm (\mathcal{N}/\mathcal{N}_{\rm eq})^p}$$

with integration constant $\mathcal{N}_{\rm eq}$, which we might expect to be O(1) .

Expectations

Away from special period $\mathcal{N} \approx \mathcal{N}_{\rm eq}$, one of the terms dominates

Monomial models

$$\epsilon(\mathcal{N}) = \frac{p}{2\mathcal{N}}$$

or (during inflation) $V(\phi) = \mu^{4-2p}\phi^{2p}$

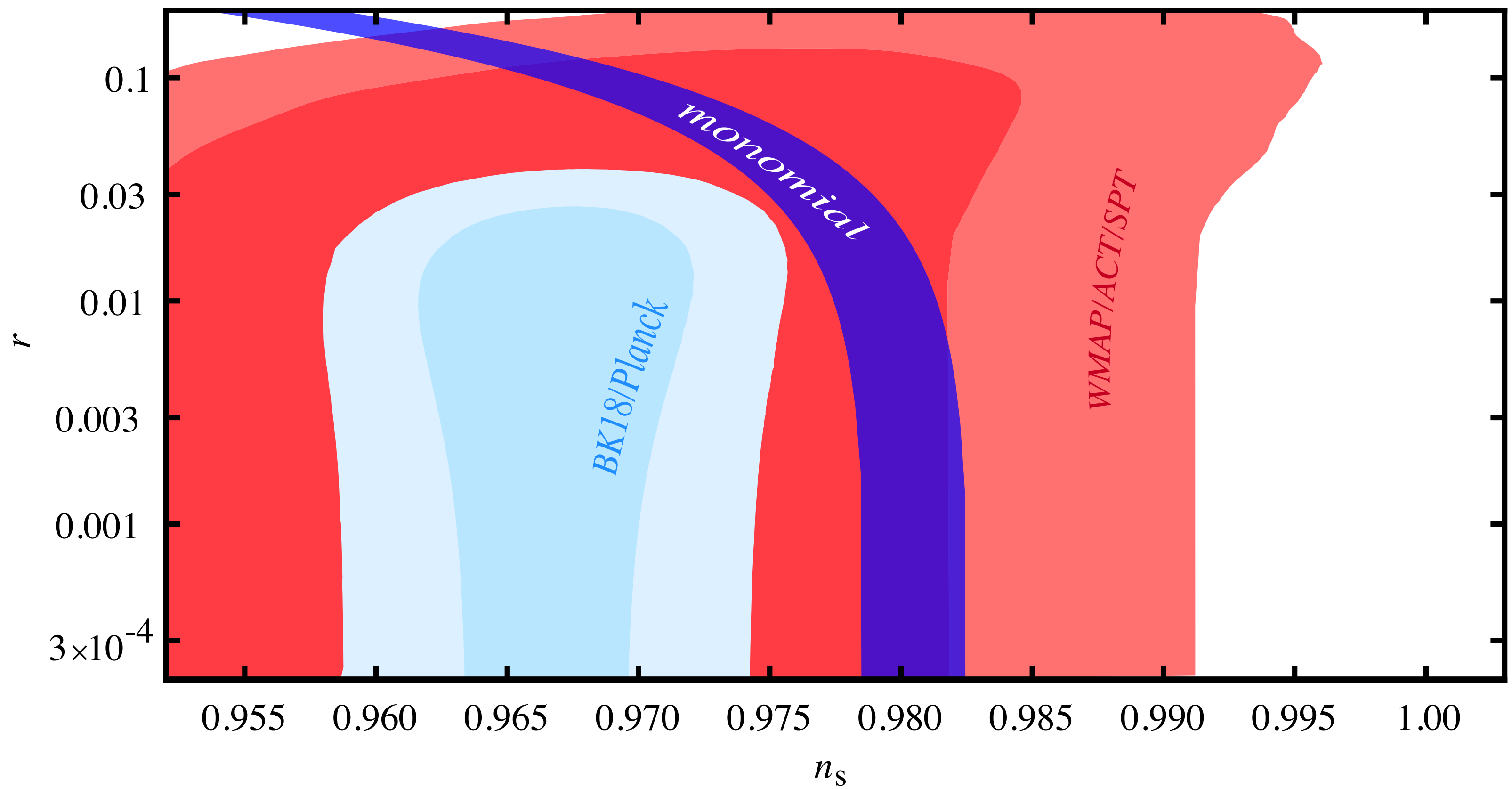
Hilltop and plateau models

$$\epsilon(\mathcal{N}) = \frac{p}{2\mathcal{N}}\left(\frac{\mathcal{N}_{\rm eq}}{\mathcal{N}}\right)^p$$

or (during inflation) $V(\phi) \approx V_0\left[1 - \left(\frac{\phi}{\Lambda}\right)^{\frac{2p}{p-1}}\right]$

Expectations

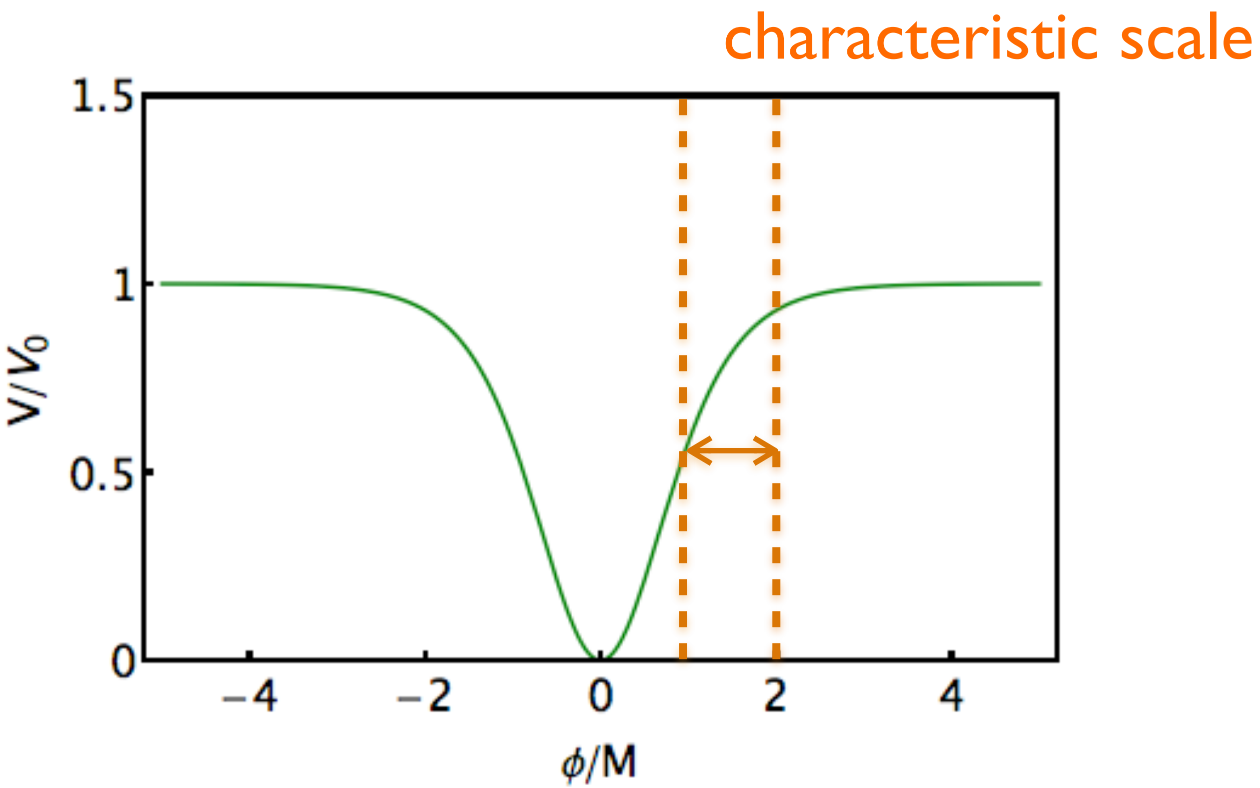
Monomial models



in their simplest form are essentially excluded by current data

Expectations

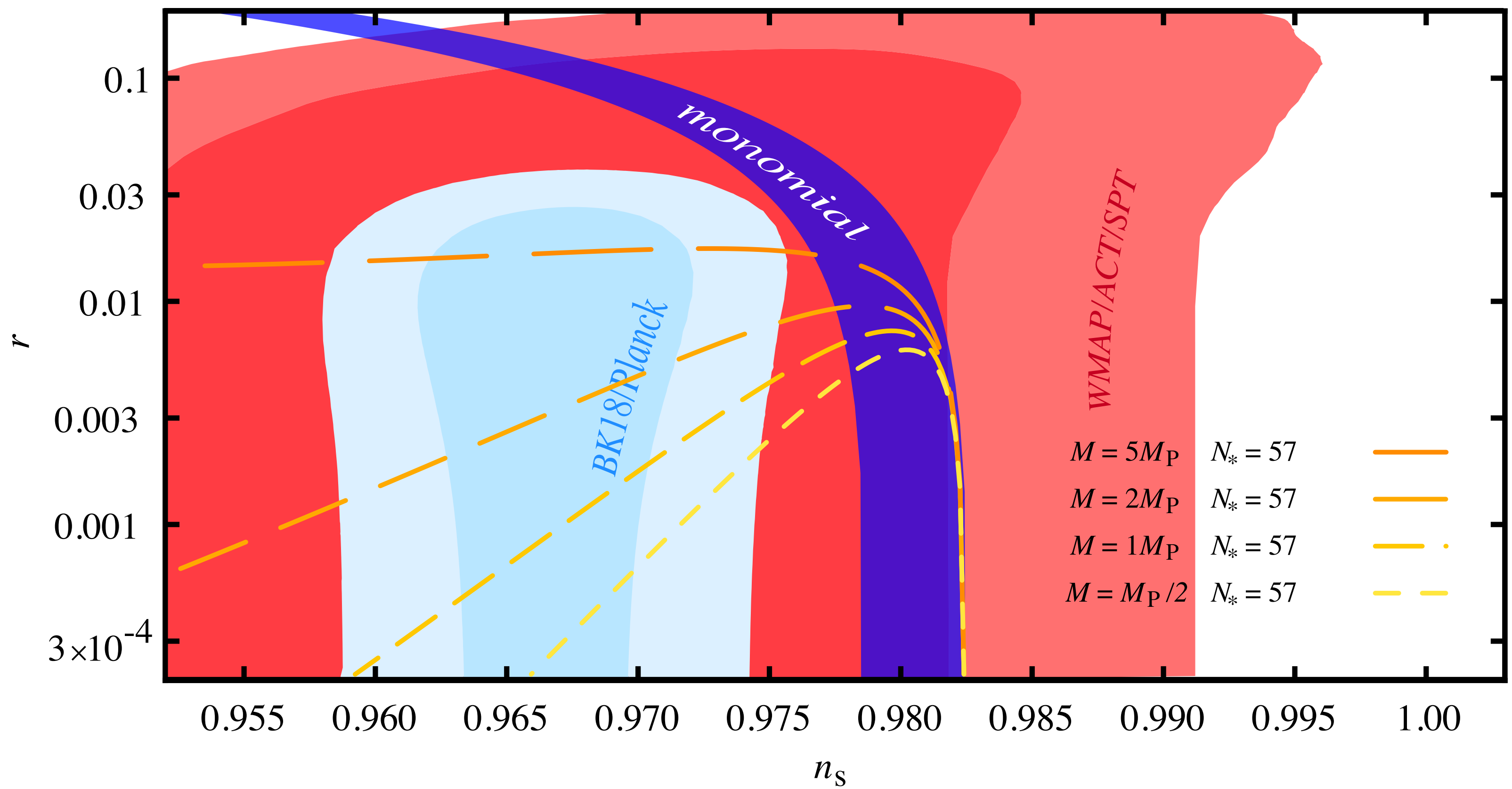
The hilltop and plateau models come with a characteristic scale over which the potential departs from a constant



The integration constant is given by $\mathcal{N}_{\rm eq} = \frac{p}{4}\left(\frac{M}{M_P}\right)^2$

Expectations

Plateau models for $M \approx M_P$



Interesting class of targets for CMB-S4

Characteristic Scale

In many models, $M \approx M_P$ because they have common origin

As an example consider Starobinsky model

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[R + \frac{1}{\mu^2} R^2 \right]$$

After Weyl rescaling

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[R - \frac{3}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \tilde{V}(\sigma) \right]$$

with $\tilde{V}(\sigma) = \frac{\mu^2}{4} (1 - e^{-\sigma})^2$

Characteristic Scale

or in terms of the canonically normalized field $\phi = M_P\sqrt{\frac{3}{2}}\sigma$

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

with $$V(\phi) = \frac{\mu^2 M_P^2}{8}\left(1 - e^{-\sqrt{2/3}\phi/M_P}\right)^2$$

Similarly for Higgs inflation, and many others the characteristic scale is set by the 4d gravitational scale.

Characteristic Scale and Beginning of Inflation

Inflation can readily start if we assume that the description of the universe by one or several scalar fields coupled to GR is valid up to the Planck scale and

$$\rho \sim \rho_{\rm grad} \sim V \sim M_{\rm P}^4$$

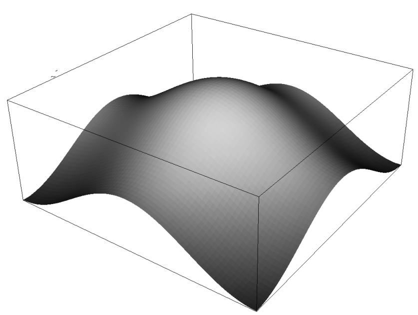
Some regions will collapse rapidly, but some will be dominated by potential energy density and inflate.

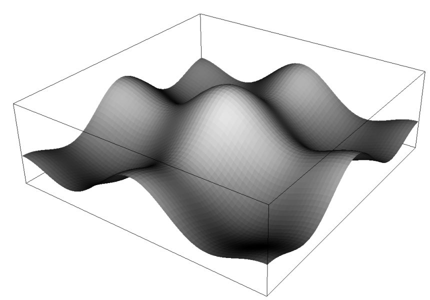
For models with $V \ll M_{\rm P}^4$, as is the case for the plateau and hilltop models, we should understand if inflation naturally arises if $\rho_{\rm grad} \gg V$.

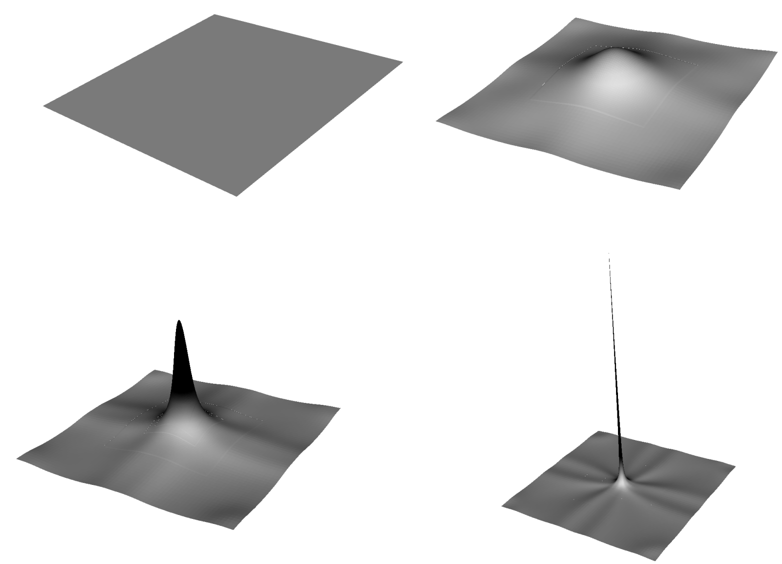
Characteristic Scale and Beginning of Inflation

Recent numerical general relativistic simulations suggest inflation is more robust to inhomogeneities than previously thought.

$$\phi(t=0,\mathbf{x}) = \phi_0 + \frac{\Delta\phi}{N}\sum_{n=1}^{N}\left(\cos\frac{2\pi n x}{L} + \cos\frac{2\pi n y}{L} + \cos\frac{2\pi n z}{L}\right)$$







If the potential is suitable over the range of field values explored, large gradients lead to black hole formation but do not inhibit inflation.

Characteristic Scale and Beginning of Inflation

Results

Monomial models are robust against inhomogeneities and inflation eventually begins even if initially $\rho_{\rm grad} \simeq 10^3 V$

Hilltop and plateau models are robust against $\rho_{\rm grad} \simeq 10^3 V$ if the field is initially confined to the plateau.

Hilltop and plateau models are robust even if the field explores the minimum provided the characteristic scale of the potential is super-Planckian $M \gtrsim M_{\rm P}$.

Inflection point models are susceptible to inhomogeneities and tuning or some mechanism is needed to set up appropriate initial conditions.

Targets

Science goals for CMB-S4

- Detect gravitational waves provided $r > 3 \times 10^{-3}$

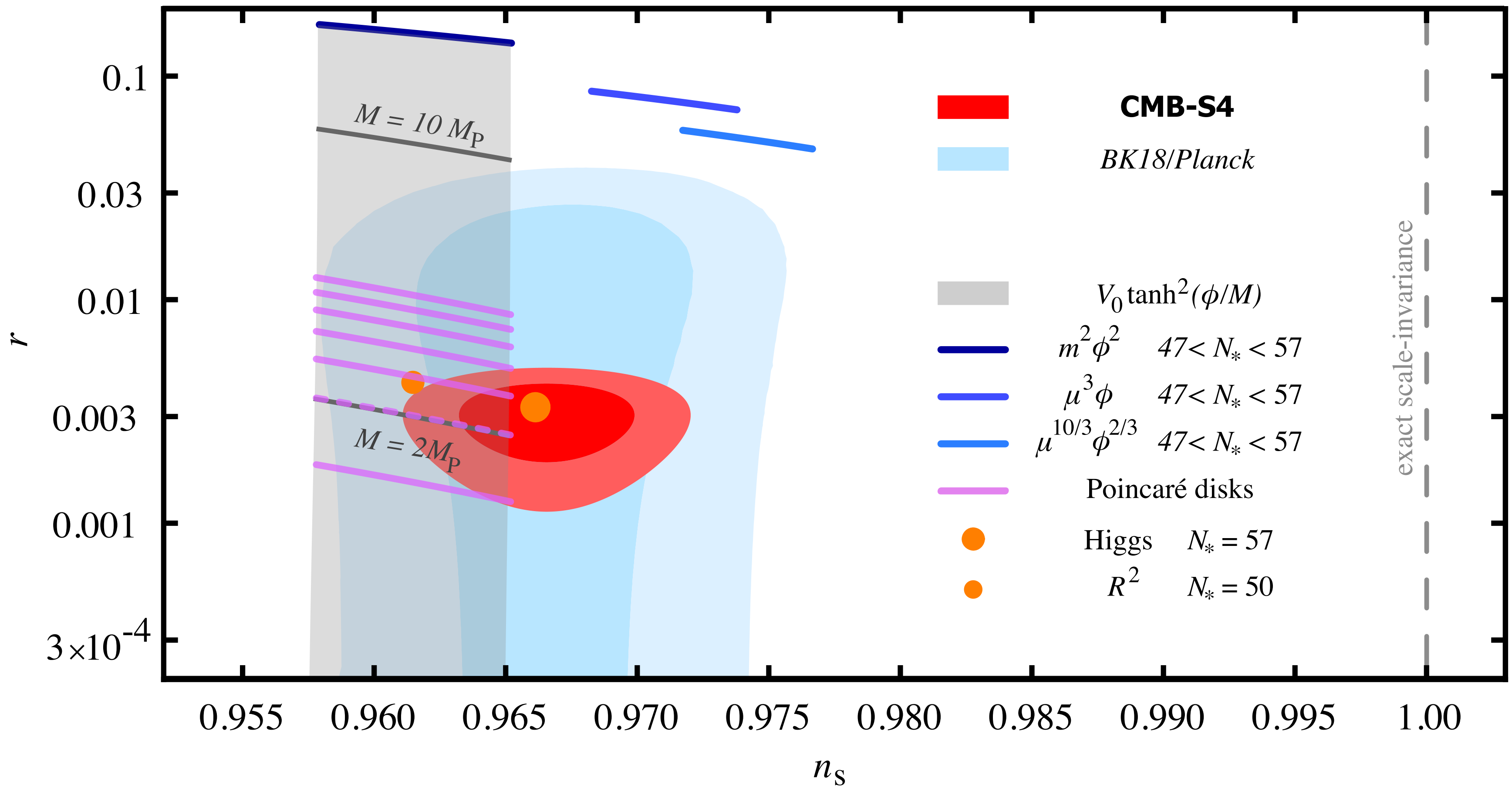
Since $M = M_P$ is an important scale and $M > M_P$
leads to $r > 10^{-3}$ for the currently preferred value of $n_{\rm s}$

- provide an upper limit of $r < 10^{-3}$ at 95% CL
  for $r = 0$

Such an upper limit would exclude all models of inflation that naturally explain the observed value of the spectral index and have a super-Planckian characteristic scale.

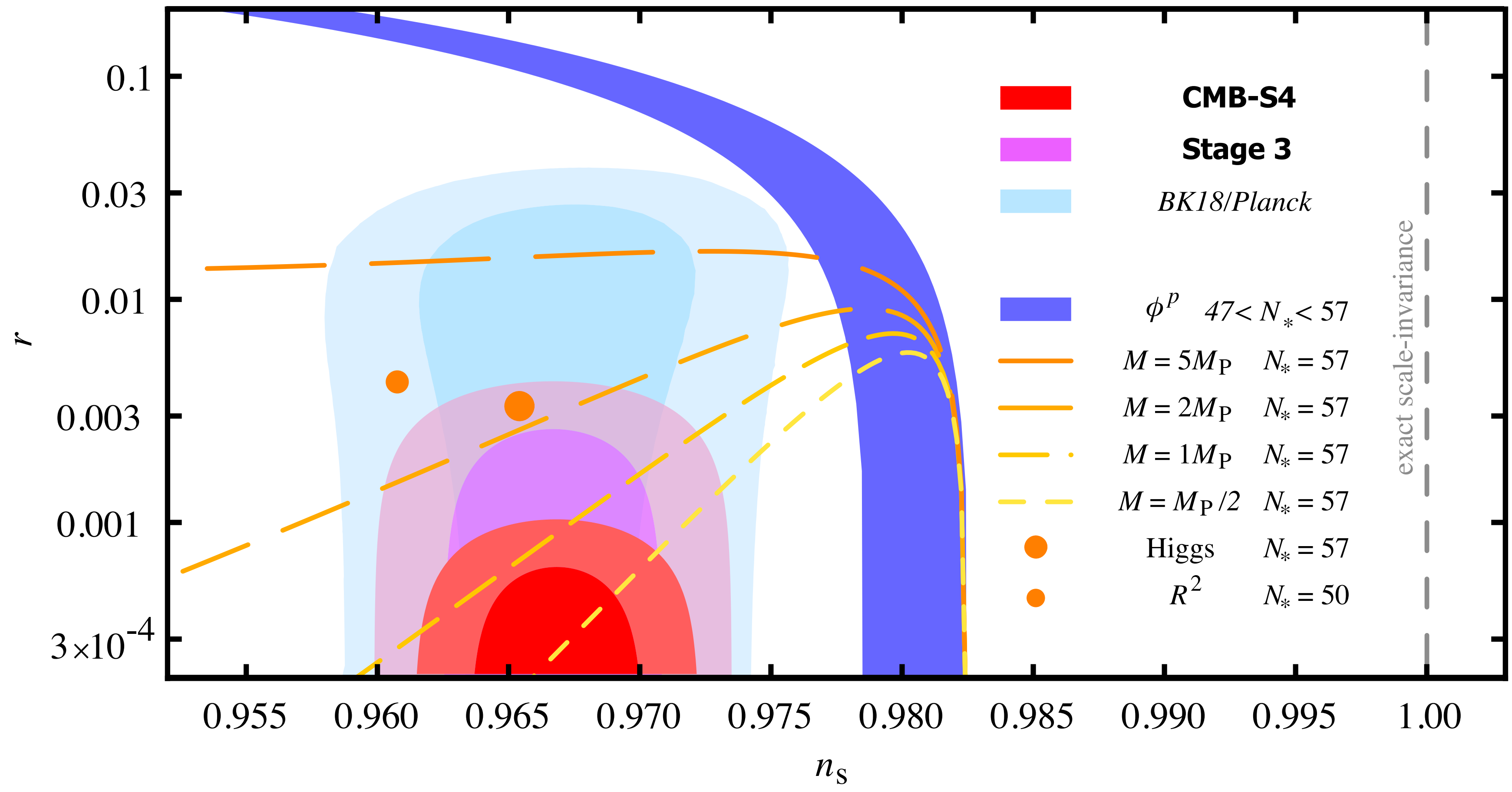
Targets

After a lot of work by a lot of people



Targets

and for an upper limit



Constraints on inflation beyond r

- What is the energy scale of inflation?
- How far did the inflaton travel?
- How did inflation begin?
- Are there additional light degrees of freedom?
- What is the propagation speed of the inflaton quanta?
- Are there additional massive spectators?
- Are there bursts of particle production?
- ...

primordial gravitational waves

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primordial gravitational waves

non-Gaussianity

Constraints on inflation beyond r

Because CMB-S4 will cross a key threshold, in the context of inflation the search for primordial gravitational waves is the key goal.

However, it will also tighten constraints on

- the scalar spectral index
- running
- features
- non-Gaussianity
- spatial curvature
- low-ℓ anomalies
- primordial magnetic fields
- cosmic strings, …

Conclusions

- With CMB-S4 we hope to detect cosmological gravitational waves present at recombination.
- Such gravitational waves would be a pristine relic of the primordial universe.
- In the foreseeable future, their imprint on the polarization of the CMB is our only way to detect them.
- These gravitational waves are independent from density perturbations and a detection would provide a new window onto the very early universe.

Conclusions

- Models of inflation based on symmetries in which inflation occurs at high energy and with large field range predict a gravitational wave signal large enough to be detected with CMB-S4.
- Even an upper limit from CMB-S4 will be extremely valuable as it would rule out or strongly disfavor all models that naturally explain the observed value of the spectral index with a trans-Planckian characteristic scale
- CMB-S4 will also significantly improve constraints on the spectral dependence, departures from adiabaticity, and non-Gaussianity of primordial density perturbations.

Thank you